LIBRATION POINT ORBITS AND APPLICATIONS

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Trajectory Correction Manoeuvres in the Transfer to Libration Point Orbits

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Statement of the problem

The **Trajectory Correction Manoeuvres (TCM)** are the manoeuvres done during the transfer segment, between the parking orbit and the target nominal one, to **compensate the velocity errors** introduced by the inaccuracies of the injection manoeuvre into the transfer orbit.

We will consider the case where the **target orbit is a libration point orbit** and the departure takes place at a fixed point near the Earth.

The main objective is the **study of the total cost** (Δv) of the TCM. We will show the results of a parametric study changing the:

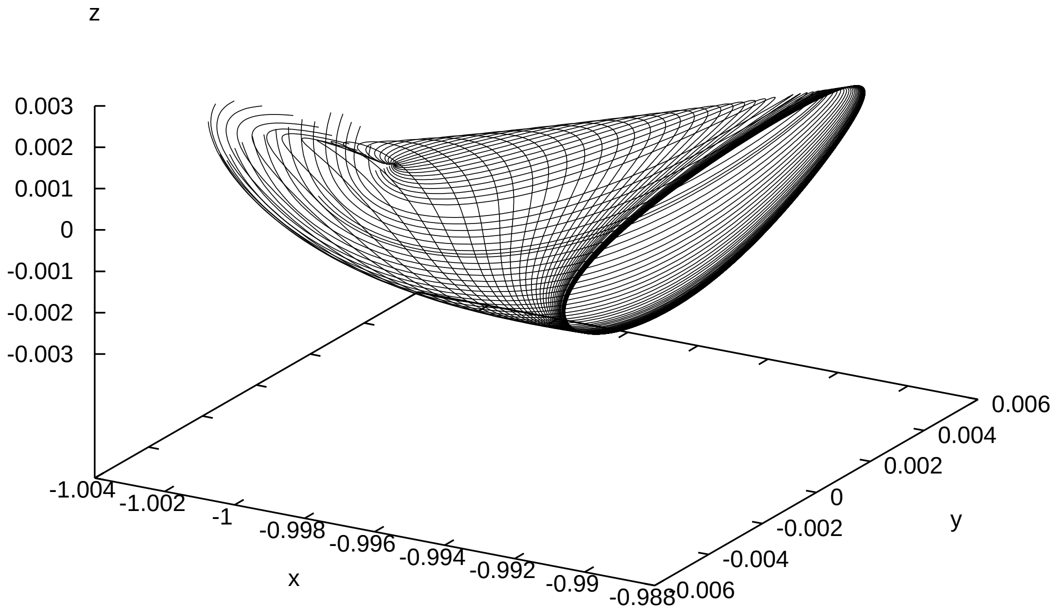
- Error in the departure velocity.
- Epoch of execution of the first TCM.
- Total time of flight.

The results will be compared with those obtained by *Serban et al. "Halo orbit mission correction manoeuvres using optimal control", Automatica 38, (2002) 571–583*, who studied the optimal TCMs in connection with the Genesis mission.

Additional **previous work** on this topic has been done by Howell & Barden (1999) (private communication).

Invariant manifolds and the transfer problem

The use of **invariant manifolds** to solve the transfer problem to libration point orbits has become a standard tool. It provides what is known as the **Dynamical Systems approach** to the problem.



In this approach, **only an insertion** manoeuvre from the parking orbit into some orbit of the stable manifold of the target orbit is required. The **final insertion manoeuvre** in the libration orbit **is not required**.

The **magnitude of the insertion manoeuvre**, Δv, is of the order of 3000 m/s (for a relatively large halo orbit with a stable manifold passing very close to the Earth). This manoeuvre is executed with some error, about a 0.2 % of the insertionΔv, which must be corrected with the TCM.

Linear and non-linear approximation of the invariant manifolds

The **stable invariant manifold** of a periodic orbit (or any other invariant object, such as an equilibrium point or a tori) is the set of points of the phase space which tend asymptotically, as time increases, to that orbit (invariant object).

If an orbit is **periodic**, the eigenvectors of the monodromy matrix give the directions of the tangent spaces (linear approximation) to the invariant manifolds at the initial condition.

Using the linear approximation it is possible to produce the **globalisation** of the manifold (stable/unstable):

- Select a distance, d, and an initial condition on the periodic orbit X. The initial conditions in the linear approximation of the manifold are given by

  $$X + d \cdot V^s(X),$$

  where $V^s(X)$ is the stable unitary eigenvector at the point X.

- Follow the above points under the flow (backwards/ forward).

In the RTBP, the stable manifold of an hyperbolic **halo orbit** is a **two dimensional manifold** (a surface on the 6-dimensional space of positions and velocities) which can be **parametrised** in the following way:

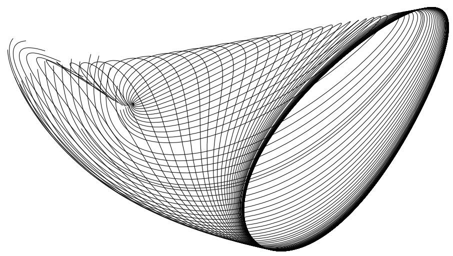
- Once a displacement d has been selected, given a **point $X^h_{s=0}$ on the halo orbit** (with period T) we can get an initial condition on the stable manifold

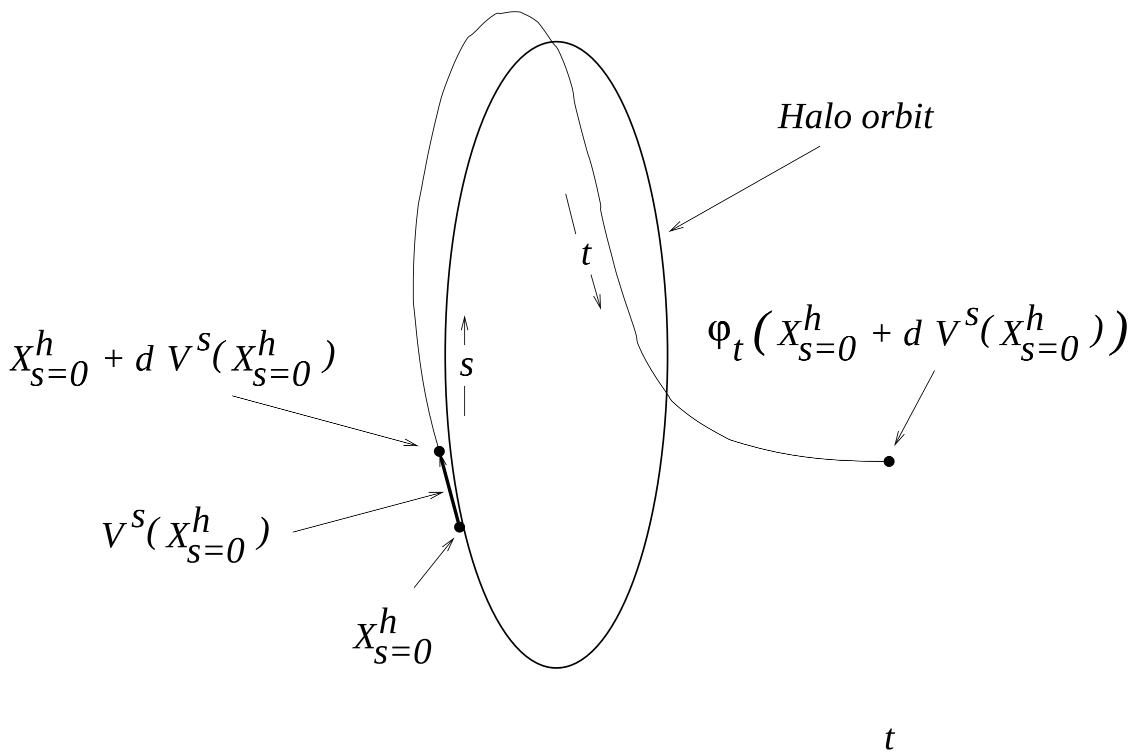
  $$X^h_{s=0} + d \cdot V^s(X^h_{s=0}).$$

  Following this point by the flow, backwards, we get all the points in the manifold associated with $X^h_{s=0}$.

  In this way X^h_s, $0 \leq s \leq T$, can be thought as one of the parameters which generate the manifold. We will call it the **parameter along the orbit**.

- The other one is the **elapsed time**, following the flow, from the initial condition $X^h + d \cdot V^s(X^h)$ to a certain point. We call this time interval the **parameter along the flow**.





Remark: This parametrisation depends on the choice of d. With a small change in d we can get the same orbits of the manifold as with a small change of X^h, only a small shift in the parameter along the flow will be observed (because the stable direction is transversal to the flow).

Non-linear approximation:

The solution of the **linearised** equations of the RTBP, around any collinear equilibrium point, can be written as

$$\begin{aligned}
x &= e^{-\lambda_{xy}t}a_s + e^{\lambda_{xy}t}a_u + \cos(\omega_{cxy}t + \phi_{xy})a_{cxy}, \\
y &= c_1 e^{-\lambda_{xy}t}a_s - c_1 e^{\lambda_{xy}t}a_u + c_2 \sin(\omega_{cxy}t + \phi_{xy})a_{cxy}, \\
z &= \cos(\omega_{cz}t + \phi_z)a_{cz},
\end{aligned}$$

If $a_s = a_u = 0$ we get libration solutions. a_s, a_u can be seen as amplitudes in the stable and unstable directions respectively. Setting $a_u = 0$, we get initial conditions for orbits in the stable manifold of a certain Lissajous orbit for this linear problem.

For the nonlinear equations, we can look for a formal series solution in terms of the four amplitudes a_s, a_u, a_{cxy}, a_{cz} and three angles $\theta_1 = \omega_1 t + \phi_{xy}$, $\theta_2 = \omega_2 t + \phi_z$, $\theta_3 = \lambda t$:

$$\left\{ \begin{array}{c} x \\ y \\ z \end{array} \right\} = \sum_{i,j=0}^{n1} \sum_{k,m=0}^{n2} \left\{ \begin{array}{c} x \\ y \\ z \end{array} \right\}_{ijkm} a_s^i a_u^j a_{cxy}^k a_{cz}^m,$$

For the halo case the solution depends on only one frequency and we must add a relation between the two central amplitudes, a_{cxy}, a_{cz}.

The coefficients x_{ijkm}, y_{ijkm}, z_{ijkm} of the formal expansions will be

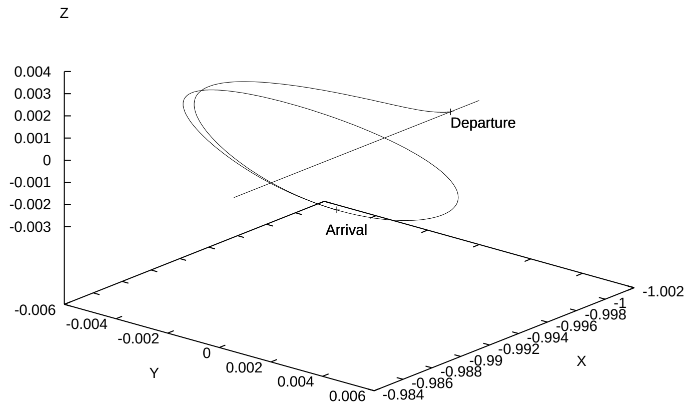
$$\left\{ \begin{array}{c} x \\ y \\ z \end{array} \right\}_{ijkm} = \sum_{p=0}^{k+m} \left\{ \begin{array}{c} xc \\ yc \\ zc \end{array} \right\}_{ijkm}^{p} e^{(i-j)\lambda t} \cos(p(\omega t + \phi)) + \left\{ \begin{array}{c} xs \\ ys \\ zs \end{array} \right\}_{ijkm}^{p} e^{(i-j)\lambda t} \sin(p(\omega t + \phi)).$$

Remark: In the halo case, the results obtained for the TCM problem, using the linear approximation and the non-linear one are the same.

The method used

We will always start with a **fixed initial condition** close to the Earth, using as reference departure point the one used by Serban et al.

The departure point is not on the stable manifold of the nominal halo orbit selected, but close to it.



The departure and "arrival" points are separated, approximately 110 days. An insertion manoeuvre at the arrival ($\Delta v \approx$13.5 m/s) is required.

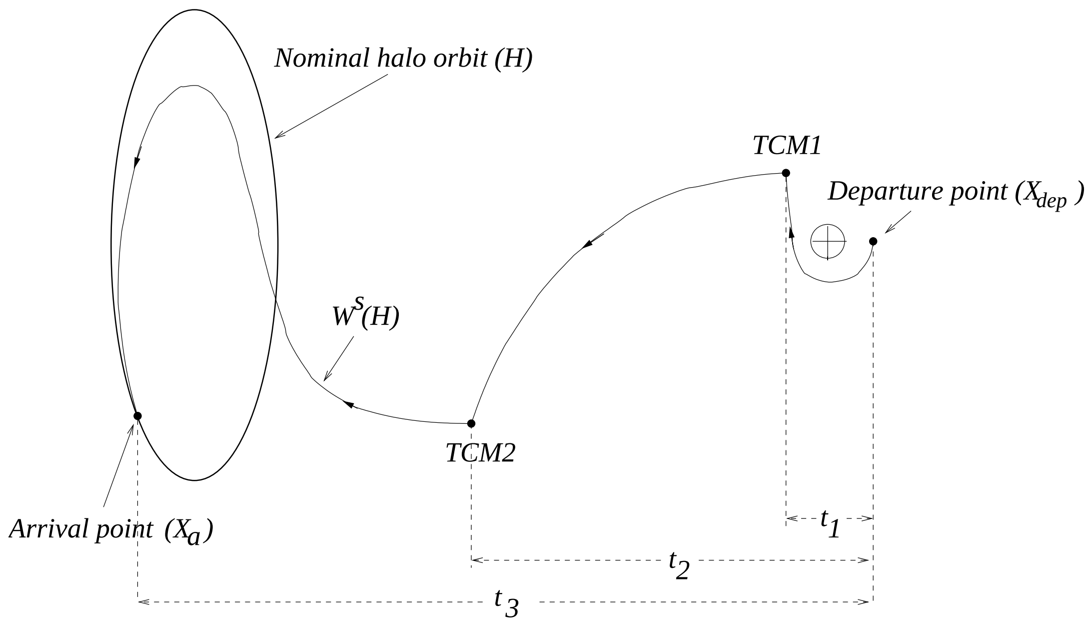
At the departure point we **modify the modulus of the velocity** according to

$$\vec{v}(0) = \vec{v}^{\,0}_{ref} \left(1 + \frac{\epsilon}{\|\vec{v}^{\,0}_{ref}\|} \right).$$

where ϵ is a parameter that we have allowed to vary between -6 m/s and $+6$ m/s.

The transfer path has **three legs**:

- The first leg goes from the fixed departure point, with the velocity affected by an error, to the point at which the first TCM is performed. This correction manoeuvre takes place a **few days after de departure**.
- The second leg, between the two trajectory correction manoeuvres TCM1 and TCM2, is used to perform the **injection on the stable manifold** of the nominal orbit.
- The last path corresponds to a piece of **trajectory on the stable manifold**. Since both TCM1 and TCM2 are assumed to be done with no errors, the spacecraft will reach the nominal halo orbit with any additional impulse.



The **epochs** of TCM1, TCM2 and arrival (at $X_a = X_a^h + d \cdot V^s(X_a^h)$) will be denoted by t_1, t_2 and t_3 respectively. The total **time of flight** will be t_3.

The **magnitudes of the corrections manoeuvre** at t_1 and t_2 will be denoted by Δv_1 and Δv_2, respectively.

The magnitude of the error, ϵ and the epoch for t_1 **will be change only in the parametric study**. If X_{dep} is the departure point

$$X_1 = \varphi_{t_1}(X_{dep}),$$

will be fixed too. ($\varphi_t(X)$ denotes the image under the flow of the RTBP of the point X after t time units).

The **equations to be solved** are

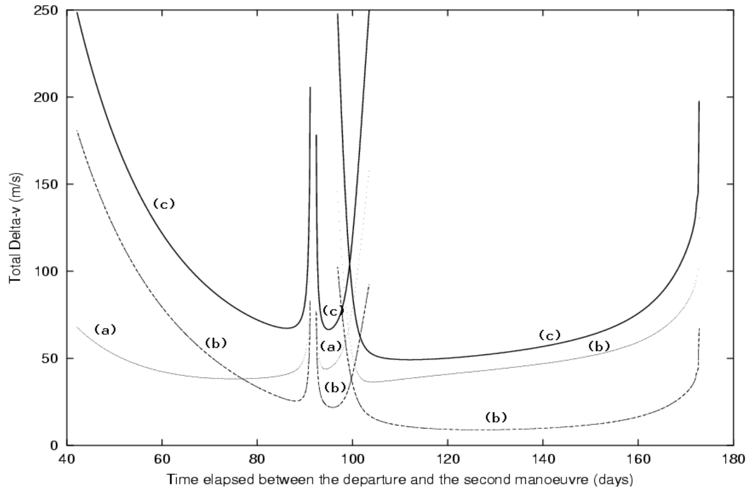
$$\varphi_{t_2-t_1}(X_1 + \Delta v_1) + \Delta v_2 \quad = \quad \varphi_{t_2-t_3}(X_a).$$

In principle, we have **six equations and fourteen unknowns**: the two manoeuvres Δv_1, Δv_2, the arrival point X_a, and the two epochs t_2 and t_3.

Eventually, some additional equations/unknowns must be added to solve the above equation with a multiple shooting procedure, or to include more that two TCMs.

In principle, there are **no privileged points on the stable manifold** to do the injection, so we will have to vary the two associated parameters (along the orbit and along the flow).

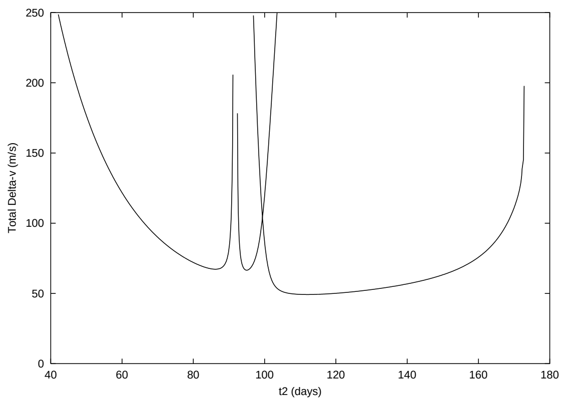
In a **first step** we have kept both parameters fixed. In this way we reduce the **number** of unknowns in our system from fourteen to **seven**. Once this initial solution is known, we have studied the variations of the arrival point (parameter along the orbit of the stable manifold) and the time of flight (substitute of the parameter along the flow).



Cost of the trajectory correction manoeuvres: The curves labelled with (a) correspond to $\|\Delta v_1\|$*, those with (b) to* $\|\Delta v_2\|$ *and those with (c) to the total cost:* $\|\Delta v_1\| + \|\Delta v_2\|$.

$\epsilon = -3$ *m/s,* $t_1 = 4$ *days,* $t_3 = 173.25$ *days,* X_a *fixed. (Optimal parameters according to Serban et al.)*

Remarks:



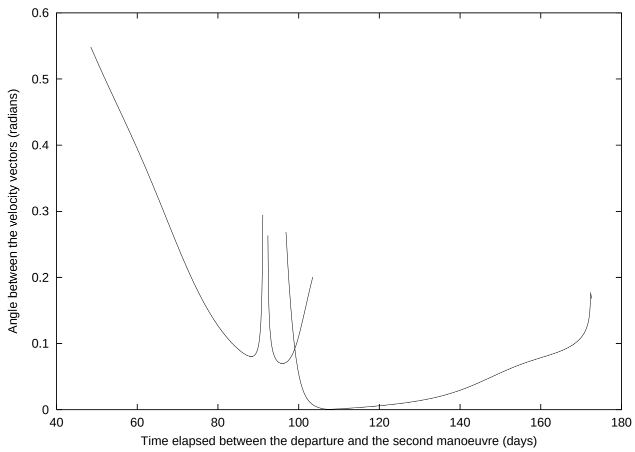
- For $t_2 = 113$ days we get the optimum solution in terms of fuel consumption: $\|\Delta v_1\| + \|\Delta v_2\| = 49.31$ m/s. This value is very close to the one given in Serban et al. (MOI approach), which is 49.1817 m/s. The discrepancies can be due to a slight difference in the nominal orbit used.

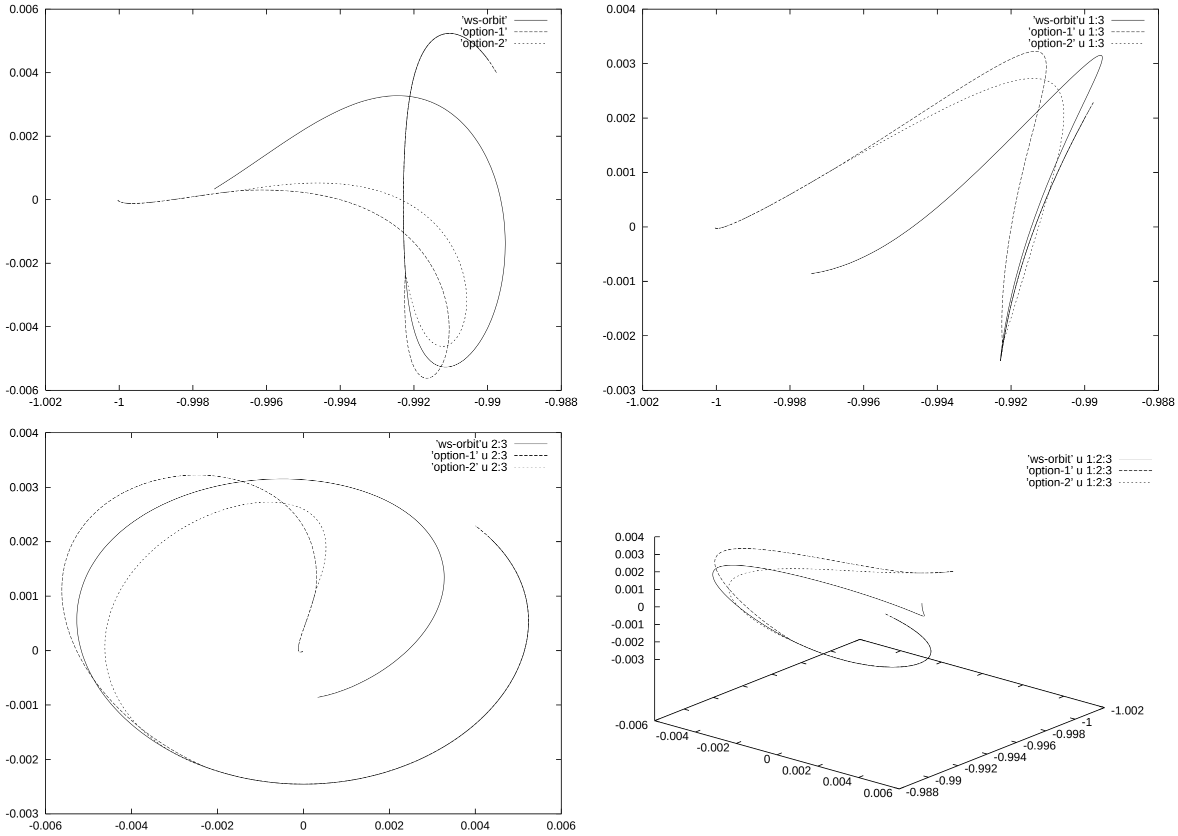
- If t_2 is small or very close to the final time (t_3) the total cost of the TCM's increases.

- The solutions are grouped in, at least, three curves. Two of them have a common point around $t_2 = 100$.

- Around the values $t_2 = 92$, 97 and 102 days, the total cost increases abruptly. This sudden grow is analogous to the one described in Howell & Barden in connection with the TCM problem for the Genesis mission. It is also similar to the one found in Wilson et al. for the cost of the transfer to a Lissajous orbit around L_2, when the time of flight between de departure and the injection in the stable manifold is fixed.

- In the figure below we show the **angle between the two velocity vectors**, as a function of t_2, along the second and third legs of the transfer path at TCM2. The qualitative behaviour of this function is analogous to the cost function. The optimal TCM correspond to an angle almost equal to zero.



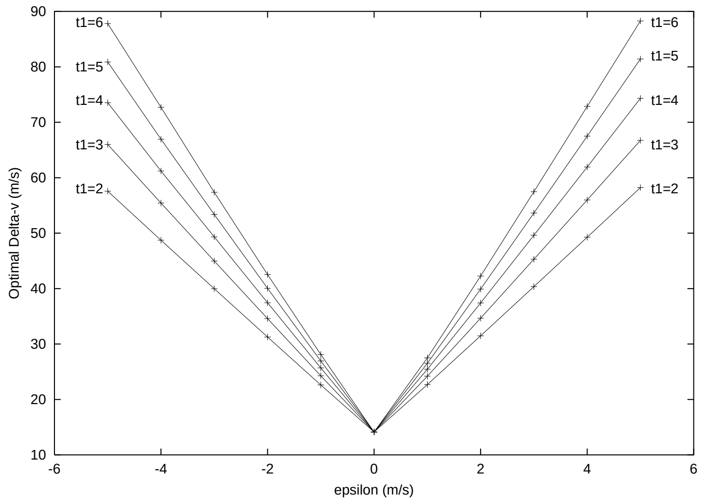


Solutions obtained for t_2= 99.5 days (double point). The cost of the TCM is the same for both (approx 100 m/s). In the figures we have included the orbit of the stable manifold at which we perform the injection.

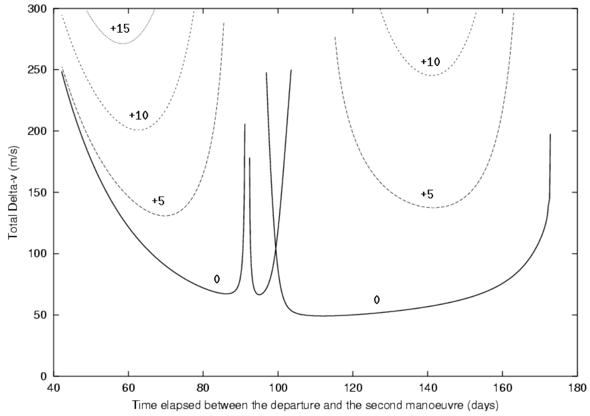
Fixing the same parameters $t_3 = 173.5$ and X_a, we proceed to a **parametric study** varying t_1 and ϵ. In the next table we give the optimal values for the TCM.

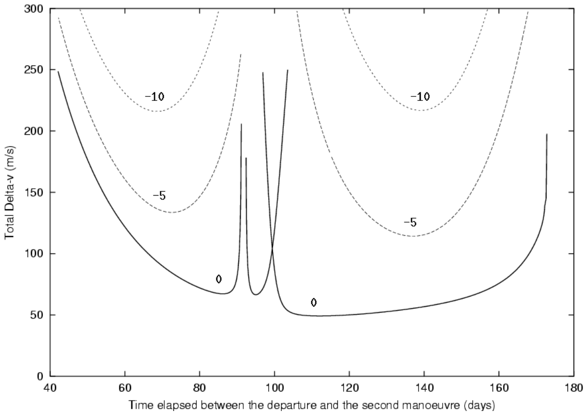
t_1	ϵ	$\|\Delta v_1\|$	$\|\Delta v_2\|$	$\|\Delta v_1\| + \|\Delta v_2\|$	Δv_{opt}
3	5	51.4551	15.2748	66.7299	
3	4	41.2822	14.7060	55.9882	
3	3	31.0891	14.1944	45.2835	
3	2	20.8642	13.7963	34.6605	
3	1	10.5610	13.6390	24.2000	
3	0	0.0974	14.0324	14.1298	
3	-1	10.8051	13.4634	24.2685	
3	-2	21.7421	12.8390	34.5811	
3	-3	32.7661	12.2004	44.9665	45.1427
3	-4	43.8878	11.5459	55.4337	55.6387
3	-5	55.1127	10.8796	65.9923	65.9416
4	5	59.0547	15.2508	74.3055	
4	4	47.4145	14.5247	61.9392	
4	3	35.7505	13.8649	49.6154	
4	2	24.0530	13.3523	37.4053	
4	1	12.2530	13.1986	25.4516	
4	0	0.0987	14.0261	14.1248	
4	-1	12.7285	12.9531	25.6816	
4	-2	25.5010	11.9398	37.4408	
4	-3	38.3502	10.9663	49.3165	49.1817
4	-4	51.3480	9.8576	61.2056	61.5221
4	-5	64.5745	8.9561	73.5306	73.4862
5	5	66.1938	15.2117	81.4055	
5	4	53.1763	14.3203	67.4966	
5	3	40.1315	13.4972	53.6287	
5	2	27.0602	12.8413	39.9015	
5	1	13.8820	12.6547	26.5367	
5	0	0.1172	14.0080	14.1252	
5	-1	14.6289	12.3175	26.9464	
5	-2	29.1862	10.8579	40.0441	
5	-3	43.8275	9.5245	53.3520	53.9072
5	-4	58.8400	8.1134	66.9534	66.8668
5	-5	74.2527	6.6145	80.8672	81.1679

The behaviour of the optimal cost with respect to ϵ and w.r.t. t_1 is linear.



Changing the parameter along the orbit (arrival point)



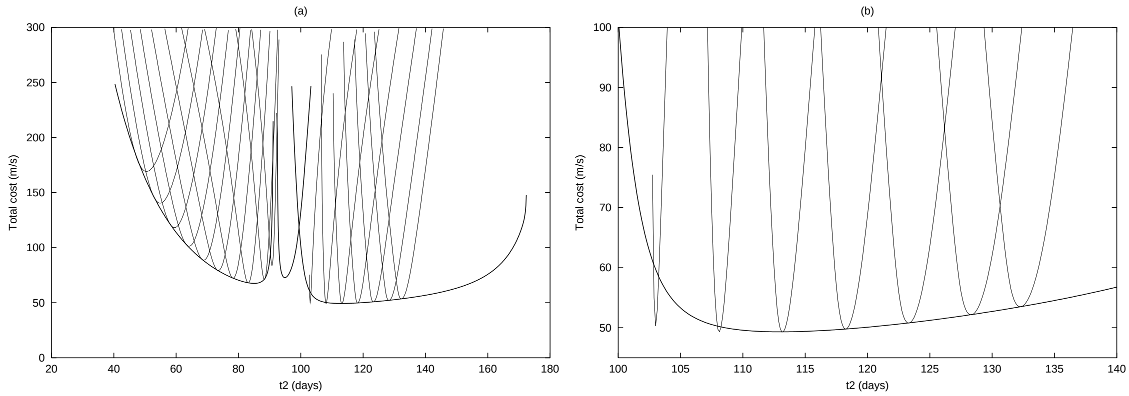


Changing the time of flight

Fixed parameters: $t_1 = 4$ days, $\epsilon =$–3 m/s and X_a.

Variable parameters: t_2 and $t_{ws} = t_3 - t_2$.

The set of solutions is two-dimensional. In the figures some sections of this set, for different (equally spaced) values of t_{ws}, are displayed.



We can remark:

- There are points (t_2, t_{ws}) for which the cost function takes values smaller than those obtained for $t_3 = 173.25$ days.

- The value of t_2 that minimises the TCM cost is a linear function of t_{ws}, with a slope close to one.

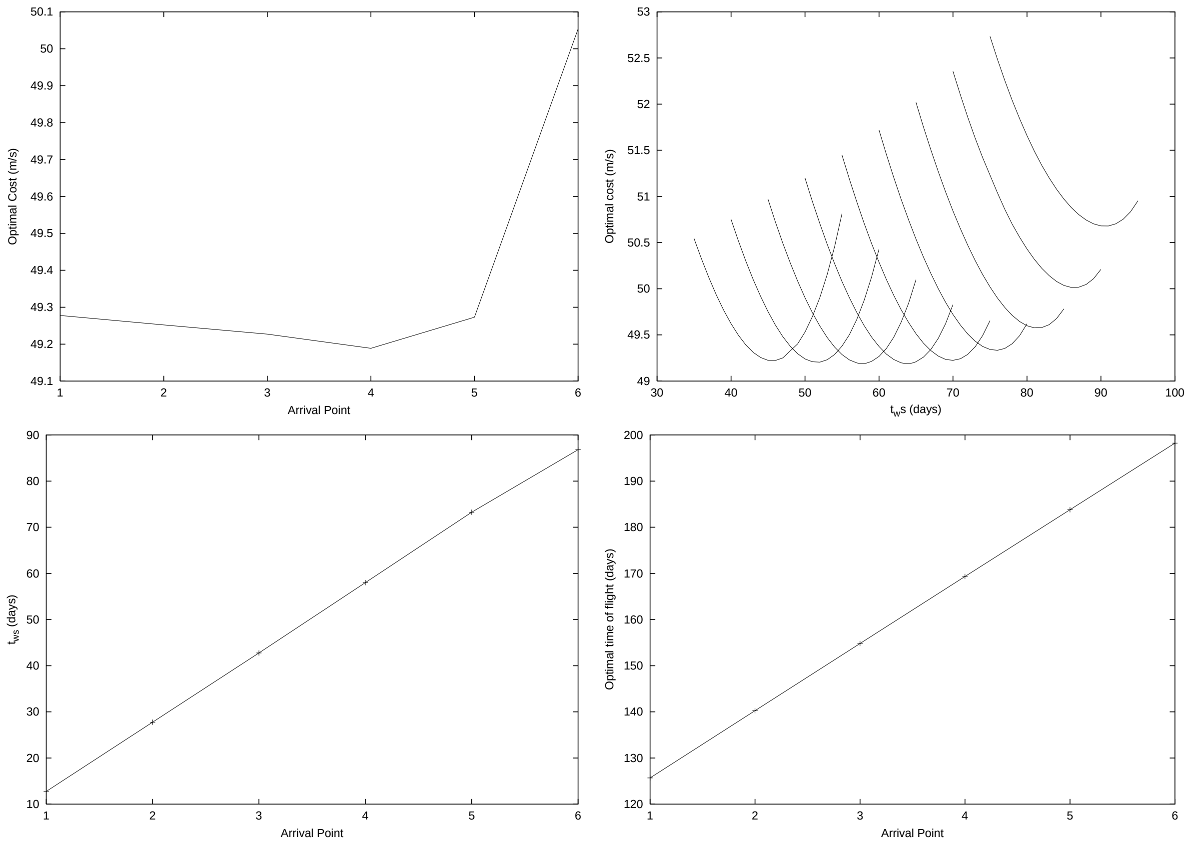
Varying the parameter along the orbit

Fixed parameters: $t_1 = 4$ days, $\epsilon = -3$ m/s.

Variable parameters: X_a, t_2 and $t_{ws} = t_3 - t_2$.

The X_a have been taken equi-distributed along the halo orbit, one every 15 days.

We just show the behaviour of the *optimal cost*, in the region close to the "best" arrival points.



The optimal solution corresponds to $t_2 = 111.14$ days, $t_3 = 172.27$ days and $\|\Delta v_1\| + \|\Delta v_2\| = 49.1861$ m/s.

Conclusions

1. The TCM problem can be studied just by using simple Dynamical Systems concepts.

2. For the optimal TCM, the results obtained with this approach agree, qualitatively and quantitatively, with those obtained with the help of Optimal control software.

3. For periodic halo orbits, the use of the linear approximation of the stable manifold gives the same results as the non linear one.

4. The developed procedure can be used for any kind of libration point orbit (under construction).